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Scheme for Cloning an Unknown Entangled State with Assistance via Non-maximally Entangled Cluster States

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Abstract We propose a scheme for cloning unknown two-particle entangled state and its orthogonal complement state with assistance from a state preparer. Two stages were included in this scheme. The first stage requires usual teleportation by using a one-dimensional non-maximally four-particle cluster state as quantum channel, after Alice's (the state sender) Bell measurement, Bob (the state receiver) can get the original state with certain probability. In the second stage, after having received Victor's (the state preparer) classical message, the perfect copies and complement copies of an unknown state can be produced in Alice's place, the probability of Alice to get the original state or its orthogonal complement state are calculated. Assisted cloning of an arbitrary unknown two-particle entangled state is discussed in the latter scheme.

Keywords Quantum cloning · Cluster state · Two-particle entangled state · Projective measurement

1 Introduction

Quantum entanglement is considered as the fundamental resource of quantum information processing such as quantum teleportation [1–3], quantum dense coding [4, 5], quantum cloning [6, 7], quantum secret sharing [8–10] and so on. Different from classical information, quantum information cannot be copied, it is so-called no-cloning theorem [11], which is a direct consequence of the linearity of quantum theory, states that it is impossible to prepare several exact copies (or clones) of an unknown quantum state. Although exact cloning is forbidden, one can design various quantum cloning machines which produce approximate clones [12–19]. For example, Bužk and Hiller was originally addressed universal quantum cloning machines [12] which is designed to generate approximate clones of states belonging

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to a finite set, Duan and Guo [13] firstly proposed the probabilistic cloning machine and so on. The other category of quantum cloning machines were developed by some authors [14–19].

Deterministic cloning and probabilistic cloning are two main kinds of the imperfect cloning. Specially, if a cloning machine performs measurements with a post selection of the measurement results, it is called probabilistic cloning. In the process of developing quantum information theory, probabilistic cloning has attracted much attention both in theoretical and experimental aspects. For this kind of the imperfect cloning, various protocol has proposed [12–24]. In 2000, Pati [20] proposed a scheme where one can produce perfect copies and orthogonal-complement copies of an arbitrary unknown state with minimal assistance from a state preparer. Recently, Chen and Wu [21] presented a protocol to probabilistically clone an unknown state and its orthogonal complement state with assistance. Zhan [22, 23] further generalize and propose a protocol where one can realize quantum cloning of an unknown bipartite entangled state and its orthogonal complement state with assistance offered by a state preparer. Very Recently, Shi and Zhan [24] propose a protocol to probabilistically clone an unknown tripartite entangled state and its orthogonal complement state with assistance from a state preparer. According to these literature, we find that almost all protocol using Bell state [20–22, 24] or GHZ state [20, 23] as the quantum channel. Hence, is it possible to assisted cloning a quantum state by using other quantum entanglement as the quantum channel? The gaol of this paper is to give a scheme for assisted cloning an unknown entangled state based on shared non-maximally four-particle cluster state. We find that the perfect copies and its orthogonal complement copies are also obtained with certain probability.

The organization of this paper is as follows. In Sect. 2, we propose a scheme which one can realize quantum cloning of an unknown two-particle entangled state and its orthogonal complement state with assistance. In Sect. 3, we propose another scheme for cloning an arbitrary unknown two-Particle entangled state with help of the state preparer.

2 Assisted Cloning of an Unknown Two-Particle Entangled State and Its Orthogonal Complement State

Suppose there are three participants, the state preparer Victor, the state sender Alice and the state receiver Bob. Alice has an unknown input two-particle entangled state $|\varphi\rangle_{12} = \alpha|00\rangle_{12} + \beta|11\rangle_{12}$ from Victor, with α as a real number and β as a complex number and $|\alpha|^2 + |\beta|^2 = 1$, $|\alpha| \geq |\beta|$. Alice wishes to help Bob to reconstruct the original state $|\varphi\rangle_{12}$ on his particles and to create either a copy or an orthogonal copy of the unknown state at her place with the assistance of Victor. The quantum channel which shared by Alice and Bob is a one-dimensional non-maximally four-particle cluster state which given by

$$|\xi\rangle_{3456} = a|0000\rangle_{3456} + b|0011\rangle_{3456} + c|1100\rangle_{3456} - d|1111\rangle_{3456}, \tag{1}$$

where the coefficients a, b, c, and d satisfy $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$ and $|a| \geq |b| \geq |c| \geq |d|$. Besides the particle pair (1, 2) from Victor, Alice has another particles 3 and 6, the other particles 4 and 5 belong to Bob. In order to help Bob to realize teleportation, Alice performs two measurement on particles (1, 3) and (2, 6) with the eigenstates [25]

$$\begin{aligned} &|00\rangle_{ij} = x|\phi^+\rangle_{ij} + y|\phi^-\rangle_{ij}, \qquad |11\rangle_{ij} = y|\phi^+\rangle_{ij} - x|\phi^-\rangle_{ij}, \\ &|10\rangle_{ij} = x|\psi^+\rangle_{ij} + y|\psi^-\rangle_{ij}, \qquad |01\rangle_{ij} = y|\psi^+\rangle_{ij} - x|\psi^-\rangle_{ij}, \end{aligned} \tag{2}$$

where x and y are real numbers, $|x|^2+|y|^2=1$ and $|x|\geq|y|$; $i,j=1,3$ and $2,6$, respectively. Hence the initial state of the combined system is

$$
\begin{aligned}
|\zeta\rangle_{123456} &= (\alpha|00\rangle_{12}+\beta|11\rangle_{12})\otimes(a|0000\rangle_{3456}+b|0011\rangle_{3456}+c|1100\rangle_{3456}-d|1111\rangle_{3456})\\
&= |\phi^+\rangle_{13}\{|\phi^+\rangle_{26}(\alpha a x^2|00\rangle-\beta d y^2|11\rangle)_{45}+|\phi^-\rangle_{26}(\alpha a x y|00\rangle+\beta d x y|11\rangle)_{45}\\
&\quad +|\psi^+\rangle_{26}(\alpha b x y|01\rangle+\beta c x y|10\rangle)_{45}+|\psi^-\rangle_{26}(-\alpha b x^2|01\rangle+\beta c y^2|10\rangle)_{45}\}\\
&\quad +|\phi^-\rangle_{13}\{|\phi^+\rangle_{26}(\alpha a x y|00\rangle+\beta d x y|11\rangle)_{45}+|\phi^-\rangle_{13}(\alpha a y^2|00\rangle-\beta d x^2|11\rangle)_{45})\\
&\quad +|\psi^+\rangle_{26}(\alpha b y^2|01\rangle-\beta c x^2|10\rangle)_{45})+|\psi^-\rangle_{26}(-\alpha b x y|01\rangle-\beta c x y|10\rangle)_{45}\}\\
&\quad +|\psi^+\rangle_{13}\{|\phi^+\rangle_{26}(\alpha c x y|10\rangle+\beta b x y|01\rangle)_{45}+|\phi^-\rangle_{26}(\alpha c y^2|10\rangle-\beta b x^2|01\rangle)_{45}\\
&\quad +|\psi^+\rangle_{26}(-\alpha d y^2|11\rangle+\beta a x^2|00\rangle)_{45}+|\psi^-\rangle_{26}(\alpha d x y|11\rangle+\beta a x y|00\rangle)_{45}\}\\
&\quad +|\psi^-\rangle_{13}\{|\phi^+\rangle_{26}(-\alpha c x^2|10\rangle+\beta b y^2|01\rangle)_{45}\\
&\quad +|\phi^-\rangle_{26}(-\alpha c x y|10\rangle-\beta b x y|01\rangle)_{45}+|\psi^+\rangle_{26}(\alpha d x y|11\rangle+\beta a x y|00\rangle)_{45}\\
&\quad +|\psi^-\rangle_{45}(-\alpha d x^2|11\rangle+\beta a y^2|00\rangle)_{45}\}.
\end{aligned}
\tag{3}
$$

From (3), one can see the probability of Alice's each result is $\frac{1}{16}$. However, corresponding to Alice's every outcomes, the state of Bob's particles can be divided into twelve main kinds. Therefore, Such special definitions are economic in consuming the classical resource in usual teleportation process. Here, suppose Alice and Bob agree the classical bits '0' to $|\phi^+\rangle_{13}|\phi^+\rangle_{26}$, '1' to $|\phi^+\rangle_{13}|\phi^-\rangle_{26}$ or $|\phi^-\rangle_{13}|\phi^+\rangle_{26}$, '00' to $|\phi^+\rangle_{13}|\psi^+\rangle_{26}$ or $|\phi^-\rangle_{13}|\psi^-\rangle_{26}$, '01' to $|\phi^+\rangle_{13}|\psi^-\rangle_{26}$, '10' to $|\phi^-\rangle_{13}|\phi^-\rangle_{26}$, '11' to $|\phi^-\rangle_{13}|\psi^+\rangle_{26}$, '000' to $|\psi^+\rangle_{13}|\phi^+\rangle_{26}$ or $|\psi^-\rangle_{13}|\phi^-\rangle_{26}$, '001' to $|\psi^+\rangle_{13}|\phi^-\rangle_{26}$, '010' to $|\psi^+\rangle_{13}|\psi^+\rangle_{26}$, '100' to $|\psi^+\rangle_{13}|\psi^-\rangle_{26}$ or $|\psi^-\rangle_{13}|\psi^+\rangle_{26}$, '110' to $|\psi^-\rangle_{13}|\phi^+\rangle_{26}$, '111' to $|\psi^-\rangle_{13}|\psi^-\rangle_{26}$ in priori. For example, if Alice's measurement outcome is $|\psi^+\rangle_{13}|\phi^-\rangle_{26}$. After performs measurement on her particles, Alice sending the classical bits '001' to Bob through a classical channel. According to Alice's classical information, Bob knows that the state of his two particles has collapsed into the state

$$|\nu\rangle_{45}=\alpha c y^2|10\rangle_{45}-\beta b x^2|01\rangle_{45}. \tag{4}$$

In this case, the resulting six-particle state can be written as

$$|\psi^+\rangle_{13}\langle\psi^+|\phi^-\rangle_{26}\langle\phi^-|\zeta\rangle=|\psi^+\rangle_{13}|\phi^-\rangle_{26}(\alpha c y^2|10\rangle_{45}-\beta b x^2|01\rangle_{45}). \tag{5}$$

In order to reconstruct the original state, Bob first operates a unitary transformation $U_1=(i\sigma_y)_4\otimes I_5$ on (4) and have

$$U_1|\nu\rangle_{45}=\alpha c y^2|00\rangle_{45}+\beta b x^2|11\rangle_{45}, \tag{6}$$

where I and $i\sigma_y$ are identity operator and the Pauli operator, respectively. Secondly, Bob introduces an auxiliary particle N with the initial state $|0\rangle_N$ and operates another unitary transformation U_2 under the basis $\{|00\rangle_{45}|0\rangle_N, |11\rangle_{45}|0\rangle_N, |00\rangle_{45}|1\rangle_N, |11\rangle_{45}|1\rangle_N\}$, namely [26]

$$U_2=\begin{pmatrix} A_1 & A_2\\ A_2 & -A_1\end{pmatrix}, \tag{7}$$

Table 1 Bob's unitary operation and the probability of successful teleportation (BST) corresponding to Alice's measurement results

AMR	U_1	$\|\tau_1\rangle$	$\|\tau_2\rangle$	BST
0	$(\sigma_z)_4 \otimes (I)_5$	$\frac{dy^2}{ax^2}$	1	$\|dy^2\|^2$
1	$(I)_4 \otimes (I)_5$	$\frac{d}{a}$	1	$\|dxy\|^2$
00	$(I)_4 \otimes (\sigma_x)_5$	$\frac{c}{b}$	1	$\|cxy\|^2$
01	$(I)_4 \otimes (i\sigma_y)_5$	$\frac{cy^2}{bx^2}$	1	$\|cy^2\|^2$
10	$(I)_4 \otimes (\sigma_z)_5$	$\frac{d}{a}$	$\frac{y^2}{x^2}$	$\|dy^2\|^2$
11	$(I)_4 \otimes (i\sigma_y)_5$	$\frac{c}{b}$	$\frac{y^2}{x^2}$	$\|cy^2\|^2$
000	$(\sigma_x)_4 \otimes (I)_5$	1	$\frac{c}{b}$	$\|cxy\|^2$
010	$(\sigma_x)_4 \otimes (i\sigma_y)_5$	1	$\frac{dy^2}{bx^2}$	$\|dy^2\|^2$
100	$(\sigma_x)_4 \otimes (\sigma_x)_5$	1	$\frac{d}{a}$	$\|dxy\|^2$
110	$(i\sigma_y)_4 \otimes (I)_5$	$\frac{y^2}{x^2}$	$\frac{c}{b}$	$\|cy^2\|^2$
111	$(i\sigma_y)_4 \otimes (\sigma_x)_5$	$\frac{y^2}{x^2}$	$\frac{d}{a}$	$\|dy^2\|^2$

where A_i is a 2×2 matrix and can expressed as

$$\begin{aligned} A_1 &= \operatorname{diag}(\tau_1, \tau_2), \\ A_2 &= \operatorname{diag}\left(\sqrt{1-\tau_1^2}, \sqrt{1-\tau_2^2}\right). \end{aligned} \tag{8}$$

Here, τ_1, τ_2 by means of the state of particles 4 and 5. Therefore, in this case, one may take $|\tau\rangle_1 = 1$, $|\tau\rangle_2 = \frac{cy^2}{bx^2}$. The unitary transformation U_2 will transform the state which described in (6) into

$$U_2U_1|\nu\rangle_{45} = cy^2(\alpha|00\rangle_{45} + \beta|11\rangle_{45}) \otimes |0\rangle_N + \beta\sqrt{b^2x^4 - c^2y^4}|11\rangle_{45} \otimes |1\rangle_N. \tag{9}$$

Then Bob measurements the particle N, if he finds $|1\rangle_N$, the teleportation is fails. If he finds $|0\rangle_N$, Bob knows the state of particles 4 and 5 has been found in the original state $|\varphi\rangle$, the maximal probability of successful teleportation is $|cy^2|^2$. About Alice's other measurement results (AMR), Bob also can obtain the original state with a certain probability by appropriate operation (see Table 1).

Next we will shown that Alice how to create either a copy or an orthogonal-complement copy of the original state $|\varphi\rangle$ with assistance of Victor. As mentioned before, if Alice applies projectors $|\psi^+\rangle_{13}\langle\psi^+|\phi^-\rangle_{26}\langle\phi^-|$ into the combined state $|\zeta\rangle$, the state of particles 1, 2, 3 and 6 will collapse in the state $|\psi^+\rangle_{13}|\phi^-\rangle_{26}$. Alice sends particles 1 and 2 to Victor and keeps particles 3 and 6 in her place. Since Victor completely knows the original state's parameters, so he can choose to measure his two particles on whatever basis. Here, Victor performs two-particle projective measurement on his particles 1, 2 in a set of mutually orthogonal basis $\{|\kappa\rangle, |\kappa_\perp\rangle, |\lambda\rangle, |\lambda_\perp\rangle\}$ which are give by

$$\begin{aligned} |\kappa\rangle &= \alpha|00\rangle + \beta|11\rangle, \qquad |\kappa_\perp\rangle = \beta^*|00\rangle - \alpha|11\rangle, \\ |\lambda\rangle &= \alpha|01\rangle + \beta|10\rangle, \qquad |\lambda_\perp\rangle = \beta^*|01\rangle - \alpha|10\rangle. \end{aligned} \tag{10}$$

The above four entangled states are related to the computation basis vectors $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, and form a complete orthogonal basis in a four-dimensional Hibert space. Obviously, the $|\kappa\rangle$ is equal to the original state and the basis $|\kappa_\perp\rangle$ is equal to $|\varphi_\perp\rangle$. Thus, the entangled state $|\psi^+\rangle_{13}|\phi^-\rangle_{26}$ in these basis can be rewritten as

$$\begin{aligned}|\psi^+\rangle_{13}|\phi^-\rangle_{26} &= |\kappa\rangle_{12}(-\beta^* x^2|01\rangle_{36} + \alpha y^2|10\rangle_{36}) + |\kappa_\perp\rangle_{12}(\alpha x^2|01\rangle_{36} + \beta y^2|10\rangle_{36})\\ &\quad + |\lambda\rangle_{12}(\beta^* xy|00\rangle_{36} - \alpha xy|11\rangle_{36}) + |\lambda_\perp\rangle_{12}(-\alpha xy|00\rangle_{36} - \beta xy|11\rangle_{36}).\end{aligned} \tag{11}$$

From (11), if Victor's measurement result is $|\kappa_\perp\rangle_{12}$ (the probability is $\frac{1}{4}$), the (3) can be rewritten as

$$\begin{aligned}&|\kappa_\perp\rangle_{12}\langle\kappa_\perp|\psi^+\rangle_{13}\langle\psi^+|\phi^-\rangle_{26}\langle\phi^-|\zeta\rangle\\ &\quad = |\kappa_\perp\rangle_{12}(\alpha x^2|01\rangle_{36} + \beta y^2|10\rangle_{36})(\alpha c y^2|10\rangle_{45} - \beta b x^2|01\rangle_{45}).\end{aligned} \tag{12}$$

Then, Victor sends the measurement outcome to Alice with two bits through a classical channel. After having received Victor's message, Alice knows that the state of her particles 3 and 6 has been found in the state $|\upsilon'\rangle_{36} = \alpha x^2|01\rangle_{36} + \beta y^2|10\rangle_{36}$. Alice first performs unitary operation $U_1' = I_3 \otimes (\sigma_x)_6$ on her particles which can transform the state $|\upsilon'\rangle_{36}$ into

$$U_1'|\upsilon'\rangle_{36} = \alpha x^2|00\rangle_{36} + \beta y^2|11\rangle_{36}. \tag{13}$$

Next Alice introduces an auxiliary particle A with the initial state $|0\rangle_A$ and makes a same unitary operation U_2' which describe in (7) under the basis $\{|000\rangle_{36A}, |110\rangle_{36A}, |001\rangle_{36A}, |111\rangle_{36A}\}$. Here, one can easily obtain $|\tau_1'\rangle = \frac{y^2}{x^2}$ and $|\tau_2'\rangle = 1$. The unitary U_2' can transform the state which describe in (13) into

$$U_2'U_1'|\upsilon'\rangle_{36} = y^2(\alpha|00\rangle_{36} + \beta|11\rangle_{36}) \otimes |0\rangle_A + \alpha\sqrt{x^2 - y^2}|00\rangle_{36} \otimes |1\rangle_A. \tag{14}$$

Finally, Alice measurement the particle A, if she finds $|1\rangle_A$, cloning of the original state fails. If her result is $|0\rangle_A$, then she can get a copy of the original state with probability $|cy^4|^2$. However, it is just a copy of the original state. If Victor's measurement outcome is $|\lambda\rangle_{12}$ and sends his result to Alice via classical channel. After obtaining two cubits from Victor, Alice knows that the state of her particles 3 and 6 has been collapse in the state $\beta^* xy|00\rangle_{36} - \alpha xy|11\rangle_{36}$. Then Alice carry out a unitary operation $I_3 \otimes I_6$ on her particles 3 and 6 to get the complement copy $|\kappa_\perp\rangle$ with probability $|cxy^3|^2$. In this case, Alice can obtain the orthogonal complement state and need not to introduce a additional particle. victor's measurement results are also maybe $|\kappa\rangle_{12}$ and $|\lambda_\perp\rangle_{12}$, respectively. In this two case, (3) can be rewritten as

$$\begin{aligned}&|\kappa\rangle_{12}\langle\kappa|\psi^+\rangle_{13}\langle\psi^+|\phi^-\rangle_{26}\langle\phi^-|\zeta\rangle\\ &\quad = |\kappa\rangle_{12}(-\beta^* x^2|01\rangle_{36} + \alpha y^2|10\rangle_{36})(\alpha c y^2|10\rangle_{45} - \beta b x^2|01\rangle_{45}),\end{aligned} \tag{15}$$

$$\begin{aligned}&|\lambda_\perp\rangle_{12}\langle\lambda_\perp|\psi^+\rangle_{13}\langle\psi^+|\phi^-\rangle_{26}\langle\phi^-|\zeta\rangle\\ &\quad = |\lambda_\perp\rangle_{12}(-\alpha xy|00\rangle_{36} - \beta xy|11\rangle_{36})(\alpha c y^2|10\rangle_{45} - \beta b x^2|01\rangle_{45}).\end{aligned} \tag{16}$$

From (15)–(16), we find that Alice also can copy of the original state and its orthogonal complement state by choosing appropriate operation. For an unknown two-particle state, our protocol produces an accurate copy of the original input state $|cy^4|^2 + |cxy^3|^2$ of the time and an orthogonal-complement copy $|cy^4|^2 + |cxy^3|^2$ of the time if Alice's measurements result is $|\psi^+\rangle_{13}|\phi^-\rangle_{26}$. In the process of teleportation, using the similar analysis, Alice will obtain a copy or a complement copy of the unknown state in her position by appropriate operation.

3 Assisted Cloning of an Arbitrary Unknown Two-Particle Entangled State and Its Orthogonal Complement State

In this section, we will shown how to cloning an arbitrary unknown two-particle entangled state by using a one-dimensional non-maximally four-particle cluster state as quantum channel. Suppose Alice has input an arbitrary unknown two-particle entangled state

$$|\varphi'\rangle_{12} = \alpha|00\rangle_{12} + \beta|01\rangle_{12} + \gamma|10\rangle_{12} + \delta|11\rangle_{12}, \tag{17}$$

where the coefficients are all complex number and satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Similarly to the cloning scheme in Sect. 2, the original state is completely unknown and arbitrary for Alice and she wishes to create either a copy or an orthogonal copy of an arbitrary unknown $|\varphi'\rangle$ at her place with the assistance of Victor. The quantum channel which shared by Alice and Bob is still a one-dimensional non-maximally four-particle cluster state which described in (1). We also assume Alice possesses particle 1, 2, 3 and 6, Bob possesses particle 4 and 5. The initial state of the combined system is

$$|\zeta'\rangle_{123456} = |\varphi'\rangle_{12} \otimes |\xi\rangle_{3456}. \tag{18}$$

In order to realize teleportation, Alice performs two measurement on particles (1, 3) and (2, 6) with the eigenstates. After Alice's measurements, all the possible outcomes are

$${}_{13}\langle\phi^+|_{26}\langle\phi^+|\zeta'\rangle = \alpha ax^2|00\rangle_{45} + \beta bxy|01\rangle_{45} + \gamma cxy|10\rangle_{45} - \delta dy^2|11\rangle_{45}, \tag{19}$$

$${}_{13}\langle\phi^+|_{26}\langle\phi^-|\zeta'\rangle = \alpha axy|00\rangle_{45} - \beta bx^2|01\rangle_{45} + \gamma cy^2|10\rangle_{45} + \delta dxy|11\rangle_{45}, \tag{20}$$

$${}_{13}\langle\phi^-|_{26}\langle\phi^+|\zeta'\rangle = \alpha axy|00\rangle_{45} + \beta by^2|01\rangle_{45} - \gamma cx^2|10\rangle_{45} + \delta dxy|11\rangle_{45}, \tag{21}$$

$${}_{13}\langle\phi^-|_{26}\langle\phi^-|\zeta'\rangle = \alpha ay^2|00\rangle_{45} - \beta bxy|01\rangle_{45} - \gamma cxy|10\rangle_{45} - \delta dx^2|11\rangle_{45}, \tag{22}$$

$${}_{13}\langle\phi^+|_{26}\langle\psi^+|\zeta'\rangle = \alpha bxy|01\rangle_{45} + \beta ax^2|00\rangle_{45} - \gamma dy^2|11\rangle_{45} + \delta cxy|10\rangle_{45}, \tag{23}$$

$${}_{13}\langle\phi^+|_{26}\langle\psi^-|\zeta'\rangle = -\alpha bx^2|01\rangle_{45} + \beta axy|00\rangle_{45} + \gamma dxy|11\rangle_{45} + \delta cy^2|10\rangle_{45}, \tag{24}$$

$${}_{13}\langle\phi^-|_{26}\langle\psi^+|\zeta'\rangle = \alpha by^2|01\rangle_{45} + \beta axy|00\rangle_{45} + \gamma dxy|11\rangle_{45} - \delta cx^2|10\rangle_{45}, \tag{25}$$

$${}_{13}\langle\phi^-|_{26}\langle\psi^-|\zeta'\rangle = -\alpha bxy|01\rangle_{45} + \beta ay^2|00\rangle_{45} - \gamma dx^2|11\rangle_{45} - \delta cxy|10\rangle_{45}, \tag{26}$$

$${}_{13}\langle\psi^+|_{26}\langle\phi^+|\zeta'\rangle = \alpha cxy|10\rangle_{45} - \beta dy^2|11\rangle_{45} + \gamma ax^2|00\rangle_{45} + \delta bxy|01\rangle_{45}, \tag{27}$$

$${}_{13}\langle\psi^+|_{26}\langle\phi^-|\zeta'\rangle = \alpha cy^2|10\rangle_{45} + \beta dxy|11\rangle_{45} + \gamma axy|00\rangle_{45} - \delta bx^2|01\rangle_{45}, \tag{28}$$

$${}_{13}\langle\psi^-|_{26}\langle\phi^+|\zeta'\rangle = -\alpha cx^2|10\rangle_{45} + \beta dxy|11\rangle_{45} + \gamma axy|00\rangle_{45} + \delta by^2|01\rangle_{45}, \tag{29}$$

$${}_{13}\langle\psi^-|_{26}\langle\phi^-|\zeta'\rangle = -\alpha cxy|10\rangle_{45} - \beta dx^2|11\rangle_{45} + \gamma ay^2|00\rangle_{45} - \delta bxy|01\rangle_{45}, \tag{30}$$

$${}_{13}\langle\psi^+|_{26}\langle\psi^+|\zeta'\rangle = -\alpha dy^2|11\rangle_{45} + \beta cxy|10\rangle_{45} + \gamma bxy|01\rangle_{45} + \delta ax^2|00\rangle_{45}, \tag{31}$$

$${}_{13}\langle\psi^+|_{26}\langle\psi^-|\zeta'\rangle = \alpha dxy|11\rangle_{45} + \beta cy^2|10\rangle_{45} - \gamma bx^2|01\rangle_{45} + \delta axy|00\rangle_{45}, \tag{32}$$

$$_{13}\langle\psi^-|_{26}\langle\psi^+|\zeta'\rangle = \alpha dxy|11\rangle_{45} - \beta cx^2|10\rangle_{45} + \gamma by^2|01\rangle_{45} + \delta axy|00\rangle_{45}, \tag{33}$$

$$_{13}\langle\psi^-|_{26}\langle\psi^-|\zeta'\rangle = -\alpha dx^2|11\rangle_{45} - \beta cxy|10\rangle_{45} - \gamma bxy|01\rangle_{45} + \delta ay^2|00\rangle_{45}. \tag{34}$$

Without lose of generality, we assume Alice's measurement outcome is $|\psi^+\rangle_{13}|\phi^-\rangle_{36}$. Then she told her result to Bob with four bits via a classical channel. According to Alice's measurement outcome, Bob knows the state of his particles has been in

$$|\omega\rangle_{45} = \alpha cy^2|10\rangle_{45} + \beta dxy|11\rangle_{45} + \gamma axy|00\rangle_{45} - \delta bx^2|01\rangle_{45}. \tag{35}$$

To achieve her goal, first, Bob carry out unitary operation $U_1 = (\sigma_x)_4 \otimes I_5$ on her particles 4 and 5, which transforms $|\omega\rangle_{45}$ into

$$U_1|\omega\rangle_{45} = \alpha cy^2|00\rangle_{45} + \beta dxy|01\rangle_{45} + \gamma axy|10\rangle_{45} - \delta bx^2|11\rangle_{45}. \tag{36}$$

Secondly, Bob introduces an auxiliary particle N in the initial state $|0\rangle_N$ and performs another unitary transformation U_2 on her particle pair (4, 5). In the basis vectors $\{|00\rangle_{45}|0\rangle_N$, $|01\rangle_{45}|0\rangle_N$, $|10\rangle_{45}|0\rangle_N$, $|11\rangle_{45}|0\rangle_N$, $|00\rangle_{45}|1\rangle_N$, $|01\rangle_{45}|1\rangle_N$, $|10\rangle_{45}|1\rangle_N$, $|11\rangle_{45}|1\rangle_N\}$, the unitary operation U_2 is a 8×8 matrix which can take the following form

$$U_2 = \begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix}, \tag{37}$$

where A_i is a 4×4 matrix and can expressed as

$$\begin{aligned} A_1 &= \mathrm{diag}(\tau_1, \tau_2, \tau_3, \tau_4), \\ A_2 &= \mathrm{diag}\Big(\sqrt{1-\tau_1^2}, \sqrt{1-\tau_2^2}, \sqrt{1-\tau_3^2}, \sqrt{1-\tau_4^2}\Big). \end{aligned} \tag{38}$$

Here, $\tau_1, \tau_2, \tau_3, \tau_4$ also depends on the state of particles 4 and 5, one may take

$$|\tau_1\rangle = \frac{d}{c}, \qquad |\tau_2\rangle = \frac{y}{x}, \qquad |\tau_3\rangle = \frac{dy}{ax}, \qquad |\tau_4\rangle = -\frac{dy^2}{bx^2}. \tag{39}$$

After Bob's collective unitary operation U_2 on her particles, the initial joint state of $U_1|\omega\rangle_{45}|0\rangle_N$ is transformed into

$$\begin{aligned} U_2U_1|\omega\rangle_{45} &= dy^2(\alpha|00\rangle_{45} + \beta|01\rangle_{45} + \gamma|10\rangle_{45} + \delta|11\rangle_{45}) \otimes |0\rangle_N + \big(y^2\alpha\sqrt{c^2-d^2}|00\rangle_{45} \\ &\quad + dy\beta\sqrt{x^2-y^2}|01\rangle_{45} + y\gamma\sqrt{a^2x^2-d^2y^2}|10\rangle_{45} \\ &\quad - y^2\delta\sqrt{b^2x^4-d^2y^4}|11\rangle_{45}\big) \otimes |1\rangle_N. \end{aligned} \tag{40}$$

At last, Bob measurement the particle N. If the measurement result is $|1\rangle_N$, Bob can't reconstruct the original state on her particles, it is means the teleportation is fails. If he finds $|0\rangle_N$, Bob can get the original state $|\varphi'\rangle$ in her position with the possibility of $|dy^2|^2$. About Alice's other measurement result (AMR), Bob's appropriate operation (include U_1 and U_2) and the probability of successful teleportation (BST) are listed in Table 2. In fact, it is easy to find that in the case of Alice's any other measurement outcomes, the probability of successful teleportation is also $|dy^2|^2$. Hence, in teleportation process, the total probability of successful teleportation is $16|dy^2|^2$.

To create either a copy or an orthogonal-complement copy of the original state $|\varphi'\rangle$, Alice also needs Victor's help. If Alice applies projector $|\psi^+\rangle_{13}\langle\psi^+|\phi^-\rangle_{26}\langle\phi^-|\psi\rangle$ into the

Table 2 Bob's unitary operation and the probability of successful teleportation corresponding to Alice's measurement results

AMR	U_1	$\vert\tau_1\rangle$	$\vert\tau_2\rangle$	$\vert\tau_3\rangle$	$\vert\tau_4\rangle$	BST
$\vert\phi^+\rangle_{13}\vert\phi^+\rangle_{26}$	$(I)_3\otimes(I)_6$	$\frac{dy^2}{ax^2}$	$\frac{dy}{bx}$	$\frac{dy}{cx}$	-1	$\vert dy^2\vert^2$
$\vert\phi^+\rangle_{13}\vert\phi^-\rangle_{26}$	$(I)_3\otimes(I)_6$	$\frac{dy}{ax}$	$-\frac{dy^2}{bx^2}$	$\frac{d}{c}$	$\frac{y}{x}$	$\vert dy^2\vert^2$
$\vert\phi^-\rangle_{13}\vert\phi^+\rangle_{26}$	$(I)_3\otimes(I)_6$	$\frac{dy}{ax}$	$\frac{d}{b}$	$-\frac{dy^2}{cx^2}$	$\frac{y}{x}$	$\vert dy^2\vert^2$
$\vert\phi^+\rangle_{13}\vert\phi^+\rangle_{26}$	$(I)_3\otimes(I)_6$	$\frac{d}{a}$	$-\frac{dy}{bx}$	$-\frac{dy}{cx}$	$-\frac{y^2}{x^2}$	$\vert dy^2\vert^2$
$\vert\phi^+\rangle_{13}\vert\psi^+\rangle_{26}$	$(I)_3\otimes(\sigma_x)_6$	$\frac{dy}{bx}$	$\frac{dy^2}{ax^2}$	-1	$\frac{dy}{cx}$	$\vert dy^2\vert^2$
$\vert\phi^+\rangle_{13}\vert\psi^-\rangle_{26}$	$(I)_3\otimes(\sigma_x)_6$	$-\frac{dy^2}{bx^2}$	$\frac{dy}{ax}$	$\frac{y}{x}$	$\frac{d}{c}$	$\vert dy^2\vert^2$
$\vert\phi^-\rangle_{13}\vert\psi^+\rangle_{26}$	$(I)_3\otimes(\sigma_x)_6$	$\frac{d}{b}$	$\frac{dy}{ax}$	$\frac{y}{x}$	$-\frac{dy^2}{cx^2}$	$\vert dy^2\vert^2$
$\vert\phi^-\rangle_{13}\vert\psi^-\rangle_{26}$	$(I)_3\otimes(\sigma_x)_6$	$\frac{dy}{bx}$	$-\frac{d}{a}$	$\frac{y^2}{x^2}$	$\frac{dy}{cx}$	$\vert dy^2\vert^2$
$\vert\psi^+\rangle_{13}\vert\phi^+\rangle_{26}$	$(\sigma_x)_3\otimes(I)_6$	$\frac{dy}{cx}$	-1	$\frac{dy^2}{ax^2}$	$\frac{dy}{bx}$	$\vert dy^2\vert^2$
$\vert\psi^-\rangle_{13}\vert\phi^+\rangle_{26}$	$(\sigma_x)_3\otimes(I)_6$	$-\frac{dy^2}{cx^2}$	$\frac{y}{x}$	$\frac{dy}{ax}$	$\frac{d}{b}$	$\vert dy^2\vert^2$
$\vert\psi^-\rangle_{13}\vert\phi^-\rangle_{26}$	$(\sigma_x)_3\otimes(I)_6$	$-\frac{dy}{cx}$	$-\frac{y^2}{x^2}$	$\frac{d}{a}$	$-\frac{dy}{bx}$	$\vert dy^2\vert^2$
$\vert\psi^+\rangle_{13}\vert\psi^+\rangle_{26}$	$(\sigma_x)_3\otimes(\sigma_x)_6$	-1	$\frac{dy}{cx}$	$\frac{dy}{bx}$	$\frac{dy^2}{bx^2}$	$\vert dy^2\vert^2$
$\vert\psi^+\rangle_{13}\vert\psi^-\rangle_{26}$	$(\sigma_x)_3\otimes(\sigma_x)_6$	$\frac{y}{x}$	$\frac{d}{c}$	$-\frac{dy^2}{bx^2}$	$\frac{dy}{ax}$	$\vert dy^2\vert^2$
$\vert\psi^-\rangle_{13}\vert\psi^+\rangle_{26}$	$(\sigma_x)_3\otimes(\sigma_x)_6$	$\frac{y}{x}$	$-\frac{dy^2}{cx^2}$	$\frac{d}{b}$	$\frac{dy}{ax}$	$\vert dy^2\vert^2$
$\vert\psi^-\rangle_{13}\vert\psi^-\rangle_{26}$	$(\sigma_x)_3\otimes(\sigma_x)_6$	$-\frac{y^2}{x^2}$	$-\frac{dy}{cx}$	$-\frac{dy}{bx}$	$\frac{d}{a}$	$\vert dy^2\vert^2$

combined state $|\zeta'\rangle$, the state of particles 1, 2, 3 and 6 will collapse in the entangled state $|\psi^+\rangle_{13}|\phi^-\rangle_{26}$. Alice sends particles 1 and 2 to Victor and keeps particles 3 and 6 in her place. Since Victor (the state preparer) completely known the parameters α, β, γ and δ of the original state $|\varphi'\rangle$, he performs a two-particle projective measurement on the qubit 1 and 2 in a set of mutually orthogonal basis vectors $\{|\chi_1\rangle, |\chi_2\rangle, |\chi_3\rangle, |\chi_4\rangle\}$, which are given as:

$$\begin{aligned}
|\chi_1\rangle &= \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle = |\varphi'\rangle,\\
|\chi_2\rangle &= \eta\alpha|00\rangle + \eta\beta|01\rangle - \eta^{-1}\gamma|10\rangle - \eta^{-1}\delta|11\rangle,\\
|\chi_3\rangle &= \beta^*|00\rangle - \alpha^*|01\rangle + \delta^*|10\rangle - \gamma^*|11\rangle,\\
|\chi_4\rangle &= \eta\beta^*|00\rangle - \eta\alpha^*|01\rangle - \eta^{-1}\delta^*|10\rangle + \eta^{-1}\gamma^*|11\rangle,
\end{aligned} \tag{41}$$

where $\eta = \sqrt{\frac{\alpha^2+\beta^2}{\gamma^2+\delta^2}}$. These four non-maximally entangled basis states $\{|\chi_1\rangle, |\chi_2\rangle, |\chi_3\rangle, |\chi_4\rangle\}$ are related to the computation basis vector $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ and form a complete orthogonal basis set in a four-dimensional Hilbert space, i.e., $\langle\chi_i|\chi_i\rangle = \delta_{ij}$. Thus, the entangled state $|\psi^+\rangle_{13}|\phi^-\rangle_{26}$ in the basis $\{|\chi_1\rangle, |\chi_2\rangle, |\chi_3\rangle, |\chi_4\rangle\}$ can be rewritten as:

$$\begin{aligned}
|\psi^+\rangle_{13}|\phi^-\rangle_{26} = {}& |\chi_1\rangle_{12}(-\beta^*xy|11\rangle_{36} + \alpha^*y^2|10\rangle_{36} - \delta^*x^2|01\rangle_{36} + \gamma^*xy|00\rangle_{36})\\
&+ |\chi_2\rangle_{12}(-\eta\beta^*xy|11\rangle_{36} + \eta\alpha^*y^2|10\rangle_{36} + \eta^{-1}\delta^*x^2|01\rangle_{36} - \eta^{-1}\gamma^*xy|00\rangle_{36})\\
&+ |\chi_3\rangle_{12}(\alpha xy|11\rangle_{36} + \beta y^2|10\rangle_{36} + \gamma x^2|01\rangle_{36} + \delta xy|00\rangle_{36})\\
&+ |\chi_4\rangle_{12}(\eta\alpha xy|11\rangle_{36} + \eta\beta y^2|10\rangle_{36} - \eta^{-1}\gamma x^2|10\rangle_{36} - \eta^{-1}\delta xy|11\rangle_{36}).
\end{aligned} \tag{42}$$

Table 3 The unitary operation corresponding to Victor's measurement results

VMR	U_1'	$\|\tau_1'\rangle$	$\|\tau_2'\rangle$	$\|\tau_3'\rangle$	$\|\tau_4'\rangle$	AS
$\|\chi_1\rangle_{12}$	$(\sigma_x)_3 \otimes (\sigma_x)_6$	$\frac{y}{x}$	1	$\frac{y^2}{x^2}$	$\frac{y}{x}$	$\|\chi_3\rangle_{12}$
$\|\chi_2\rangle_{12}$	$(\sigma_x)_3 \otimes (\sigma_x)_6$	$\frac{y}{x}$	1	$\frac{y^2}{x^2}$	$\frac{y}{x}$	$\|\chi_4\rangle_{12}$
$\|\chi_4\rangle_{12}$	$(\sigma_x)_3 \otimes (\sigma_x)_6$	$\frac{y}{x}$	1	$\frac{y^2}{x^2}$	$\frac{y}{x}$	$\|\chi_2\rangle_{12}$

If the result of Victor's measurement on the two particles 1 and 2 is $|\chi_3\rangle_{12}$, (3) can be written as:

$$\begin{aligned}&|\chi_3\rangle_{12}\langle\chi_3|\psi^+\rangle_{13}\langle\psi^+|\phi^-\rangle_{26}\langle\phi^-|\zeta'\rangle\\&\quad=|\chi_3\rangle_{12}(\alpha xy|11\rangle_{36}+\beta y^2|10\rangle_{36}+\gamma x^2|01\rangle_{36}+\delta xy|00\rangle_{36})(\alpha cy^2|10\rangle_{45}\\&\qquad+\beta dxy|11\rangle_{45}+\gamma axy|00\rangle_{45}-\delta bx^2|01\rangle_{45}).\end{aligned}\tag{43}$$

Victor sends the measurement outcome to Alice through classical channel with two classical bits. After having received Victor's message, Alice knows her particle has been in the state

$$|\omega'\rangle_{36}=\alpha xy|11\rangle_{36}+\beta y^2|10\rangle_{36}+\gamma x^2|01\rangle_{36}+\delta xy|00\rangle_{36}.\tag{44}$$

In order to cloning the original state, Alice first performs unitary operation $U_1'=(\sigma_x)_3\otimes(\sigma_x)_6$ on his particles 3 and 6, then we can get

$$U_1'|\omega'\rangle_{36}=\alpha xy|00\rangle_{36}+\beta y^2|01\rangle_{36}+\gamma x^2|10\rangle_{36}+\delta xy|11\rangle_{36}.\tag{45}$$

After carry out the operation U_1', Alice introduces an auxiliary particle A in the initial state $|0\rangle_A$ and performs another unitary transformation U_2' which describe in (37), here,

$$|\tau_1'\rangle=\frac{y}{x},\qquad|\tau_2'\rangle=1,\qquad|\tau_3'\rangle=\frac{y^2}{x^2},\qquad|\tau_4'\rangle=\frac{y}{x}.\tag{46}$$

Then, the initial joint state which describe in (45) is transformed into

$$\begin{aligned}U_2'U_1'|\omega'\rangle_{36}&=y^2(\alpha|00\rangle_{36}+\beta|01\rangle_{36}+\gamma|10\rangle_{36}+\delta|11\rangle_{36})\otimes|0\rangle_A+(\alpha y\sqrt{x^2-y^2}|00\rangle_{36}\\&\quad+\gamma\sqrt{x^4-y^4}|10\rangle_{36}+\delta y\sqrt{x^2-y^2}|11\rangle_{36})\otimes|1\rangle_A.\end{aligned}\tag{47}$$

Finally, Alice measures the particle A in the bases $\{|0\rangle_A,|1\rangle_A\}$. If the measurement result is $|0\rangle_A$, Alice knows she has already successfully reconstructed the original state on her particle 3 and 6. Otherwise, the scheme fails. Evidently, the maximal probability of successful cloning the original state is $|dy^4|^2$ if Alice's measurement result is $|\psi^+\rangle_{13}|\phi^-\rangle_{26}$. However, it is just a copy of the original arbitrary unknown state. Surely, it is also possible for Alice to get the state $|\chi_1\rangle_{12}$, $|\chi_2\rangle_{12}$, $|\chi_4\rangle_{12}$. If so, the state of the particle 3 and 6 will collapse to the state which describe in (42). If Victor sends her measurement result (VMR) to Alice, Alice can get the orthogonal complement state (AS) by appropriate operation (see Table 3).

Based on above analysis, Alice can get an arbitrary unknown two-particle entangled state and its orthogonal complement state with certain probability. For an arbitrary unknown two-particle state, the probability of Alice produces an accurate copy of the original input state is $|dy^4|^2$ and get an orthogonal-complement copy is $3|dy^4|^2$ if Alice's measurement result is

$|\psi^+\rangle_{13}|\phi^-\rangle_{26}$. In the process of teleportation, if Alice's measurement outcome is one of the other 15 states (i.e., $|\phi^\pm\rangle_{26}|\phi^\pm\rangle_{13}$, $|\phi^\pm\rangle_{26}|\psi^\pm\rangle_{13}$, $|\psi^\pm\rangle_{26}|\phi^+\rangle_{13}$, $|\psi^\pm\rangle_{26}|\psi^\pm\rangle_{13}$), applying the same analysis method as above, Alice also can obtain a copy or complement copy of the arbitrary unknown state at her place.

4 Conclusion

In summary, We have proposed a scheme that one can produce perfect copies or orthogonal complement copies of an unknown two-particle entangled state via quantum and classical channel with help of a state preparer. The cloning scheme needs two step. In the first step, Alice needs to performs measurement on particles with the eigenstates and seeds her measurement results to Bob via classical channel. Based on Alice's classical information, Bob can reconstruct the original state on his particles. However, the particles 1, 2, 3 and 6 are still in Alice's place and the quantum channel is non-maximally entangled state, so it is a probabilistic teleportation process in essence. In the second step, Victor performs two-particle projective measurement on the particle which seeded by Alice. After having received Victor's measurement result through classical channel, Alice can get the input entangled state and its orthogonal complement state by appropriate unitary operation. The probability of successful teleportation and cloning the original entangled state and its orthogonal complement state are also calculated. Furthermore, our scheme can be easily generalized to producing more copies or complement copies, it is also can generalized to an unknown N-particle entangled state case.

Different from other protocol [20–24], our protocol choosing one-dimensional non-maximally four-particle cluster state which can not being reducible to a tensor product of two Bell states as the quantum channel. Moreover, the cluster states have the properties both of the GHZ-class and the W-class entangled states in the case of $N > 3$ and it is harder to be destroyed by local operations than GHZ-class states [27]. Except four-particle cluster state, our protocol also need Bell-state measurement, classical communication, and two-particle projective measurement. These are all feasible according to the present technologies. In optical systems, a four-qubit cluster state has been prepared and applied to the Grover search algorithm [28, 29]. Nowadays, the cluster state has been the universal resource in one-way quantum computation [30] and a number of feasible protocols for generating entangled four-particle cluster states have been proposed [31–34], so our cloning protocol may be helpful to realized the cluster state's potential characteristic and the protocol may be realized in the realm of current experimental technology.

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